1882MNRAS..42..446Z

The nucleus was well defined and bright, like a star of the second magnitude, the tail spreading out like a fan to the extent of about two and a half degrees of altitude, as visible with the naked eye. The tail stretched upwards from the horizon, with a slight curve in it towards the southward, at the upper end.

The comet remained visible until it sank behind the clouds on the horizon. Careful search was made for it the next night, but without success. We did not see it again, even with the aid of a telescope.

On our arrival at Hong Kong on June 30 we made inquiries about it, but found that nobody had seen it or heard anything about it until we ourselves reported it.

The Solar Eclipse of 1882, *May* 17, *observed at Vizagapatam.*

By A. V. Nursinga Row, Esq.

The partial eclipse of the Sun of 1882, May 17, was observed at the Observatory, Daba Gardens, Vizagapatam (Latitude 17° 42′ 9″ N., Longitude $5^h\ 33^m\ 32^s{\cdot}3$ E., according to the Trigonometrical Survey). There were no clouds, but the sky was hazy, and the Sun was surrounded by a halo during the whole time of the eclipse. The beginning of the eclipse was observed by observer Verabadroodoo at local sidereal time $4^h\ 25^m\ 17^s$; the end was observed by Mr. A. V. Juggarow at local sidereal time $6^h\ 57^m\ 21^s$.

On the Solution of Kepler's Problem. By Prof. Ch. V. Zenger.

The various modes of calculating the true anomaly by means of the mean anomaly and eccentricity of the planetary orbit considered by Mädler, Leverrier, and J. W. L. Glaisher, require a considerable amount of calculation, partly owing to the slow convergence of the series employed.

Writing

$$E - e \sin E = F = m\,(T - t),$$

where E denotes the true anomaly, e the eccentricity, m the mean anomaly, and T the time of perihelion, in the following manner—

$$E - F = e \sin E,$$

it becomes obvious, from the small amount of the eccentricity, that E—F will be always small, and the series will rapidly converge.

Now

$$E - F = \sin(E-F) + \frac{1}{6}\sin^3(E-F) + \frac{3}{40}\sin^5(E-F) + \frac{15}{336}\sin^7(E-F) + \text{\&c.};$$

and we thus have

$$\frac{\sin(E-F)}{\sin E} = \frac{e}{1 + \frac{1}{6}\sin^2(E-F) + \frac{3}{40}\sin^4(E-F) + \frac{15}{336}\sin^6(E-F) + \ldots}$$

1882MNRAS..42..446Z

and therefore

$$\cot F - \cot E = \frac{e \operatorname{cosec} F}{1 + \frac{1}{6}\sin^2(E-F) + \frac{3}{40}\sin^4(E-F) + \frac{15}{336}\sin^6(E-F)}.$$

For the first approximation E − F may be replaced by its maximum value, viz.

$$E - F = ew \sin 90^\circ = ew,$$

where ew is the angular value of the eccentricity; the first approximation thus is

$$\cot E = \cot F - \frac{e \operatorname{cosec} F}{1 + \frac{1}{6}\sin^2 ew + \frac{3}{40}\sin^4 ew + \frac{15}{336}\sin^6 ew}.$$

The value thus obtained gives the angular difference

$$E - F = e \sin E.$$

We have also

$$dE = \frac{dF}{1 - e \cos E},$$

and correcting by the known variation of E, we get immediately with the first approximation, a value correct to the seconds, as the following parallel calculations of the true anomaly of *Mars* by Mädler's and the present method show, without the application of the *regula falsi*.

Mädler's method.—Suppose the true anomaly of *Mars* required for

$$\text{April } 24,\ 13^h\ 25^m\ 15^s{\cdot}0,$$

$$\frac{T = \text{Jan. } 8,\quad 9^h\ 44^m\ 0^s{\cdot}0}{T - t = \quad 107,\quad 3^h\ 41^m\ 15^s{\cdot}0},$$

$$m = \frac{360^\circ}{686{\cdot}97964} = 0^\circ\ 31'\ 26''{\cdot}516,$$

$$m(T - t) = F = 56^\circ\ 9'\ 7''{\cdot}4,\ e = 0{\cdot}0932168,$$

$$ew = 19227''{\cdot}24 = 5^\circ\ 20'\ 27''{\cdot}04.$$

Mädler says the value of E must be comprised between 59° 9′·12 and 61° 29′·55, and puts E=60° for the first approximation,

$$\log \sin 60^\circ = 9{\cdot}9375306,$$

$$\log ew = \frac{4{\cdot}28839195}{4{\cdot}2214501} = \log 16651''{\cdot}53,$$

$$E - ew \sin E = 60^\circ - 4^\circ\ 37'\ 31''{\cdot}33 = \quad 56^\circ\ 22'\ 28''{\cdot}67,$$

$$\frac{F = \quad 56^\circ\ 9'\ 7''{\cdot}40}{dF = -\quad 0^\circ\ 46'\ 38''{\cdot}73}.$$

For comparison, I now give the first approximation obtained by my own method.

1882MNRAS..42..446Z

$$\cot E = 0{\cdot}670645 - \frac{0{\cdot}112243}{1 + \frac{\sin^2(5^\circ\ 20'{\cdot}45)}{6} + \frac{3\sin^4(5^\circ\ 20'{\cdot}45)}{40}},$$

$$\cot E = 0{\cdot}670645 - \frac{0{\cdot}112243}{1{\cdot}01448} = 0{\cdot}670645 - 0{\cdot}110390,$$

$$\cot E = 0{\cdot}560255,\quad E = 60^\circ\ 45'\ 36'',$$

$$\begin{aligned} \log\sin 60^\circ\ 45'\ 36'' &= 9{\cdot}94079,\quad F' = ew = 4^\circ\ 39'\ 35''{\cdot}6,\\ \log ew &= \underline{4{\cdot}28392}\\ &\ 4{\cdot}22471 = \log 16775''{\cdot}6, \end{aligned}$$

$$\begin{aligned} E - F' = 60^\circ\ 45'\ 36''{\cdot}0 - 4^\circ\ 39'\ 35''{\cdot}6 &= 56^\circ\ 6'\ 0''{\cdot}4,\\ F &= \underline{56^\circ\ 9'\ 7''{\cdot}4}\\ \Delta F' &= \ 0^\circ\ 3'\ 7''{\cdot}0' \end{aligned}$$

$$\Delta F : \Delta F' = 46'{\cdot}645 : 3'{\cdot}12 = 15 : 1.$$

The first approximation by my own method is fifteen times better than Mädler's result.

But using the differential relation

$$dE = \frac{187''}{1 - 0{\cdot}09322\cos 60^\circ\ 45'{\cdot}6} = 195''{\cdot}92,$$

the corrected true anomaly will be

$$\begin{aligned} E + \delta E = E_{,} &= 60^\circ\ 45'\ 36''{\cdot}0 + 3'\ 15''{\cdot}92,\\ E_{,} &= 60\ \ 48\ \ 51{\cdot}92. \end{aligned}$$

Mädler has to go through three approximative calculations to find finally the true anomaly

$$E = 60^\circ\ 48'\ 53''{\cdot}78,$$

giving an error

$$\Delta F = 0\ \ 0\ \ 0{\cdot}04.$$

It is obvious how rapidly the proposed method, using only one approximation, gives to 1″·80 the true value of E.

The next approximation would bring the value of E to

$$60^\circ\ 48'\ 53''{\cdot}72.$$

Prague:
1882, *July* 17.

Postscript.

The largest planetary eccentricity not exceeding 0·4, it will in all cases be sufficient to take the two first terms of powers of sin (E − F), which may be replaced for the first approximation by

$$\theta_0 = 1 + \frac{1}{6}\sin^2(ew) + \frac{3}{40}\sin^4(ew).$$

1882MNRAS..42..446Z

It will greatly shorten computation if θ_0 be given by a small table for the eight principal planets.

The following table gives for those planets the terms log θ_0 and log $\frac{e}{\theta_0}$:

I. log θ_0 and log $\left(\frac{e}{\theta_0}\right)$.

	e	ew	θ_0	log θ_0	log $\frac{e}{\theta_0}$	log e
☿	0·2056048	11° 46′ 55″·89	1·082021	0·0342357	9·2787978	9·3130335
♀	0·0068433	0 23 31·53	1·000008	0·0000037	7·8352619	7·8352656
♁	0·0167701	0 57 39·08	1·000047	0·0002041	8·2243316	8·2245357
♂	0·0932611	6 43 41·21	1·002288	0·0009882	8·9687025	8·9697007
♃	0·0482388	2 45 49·97	1·000388	0·0001685	8·6832281	8·6833966
♄	0·0559956	2 55 49·92	1·000436	0·0001893	8·7479645	8·7481538
♅	0·0465775	2 40 7·30	1·000361	0·0001568	8·6680194	8·6681762
♆	0·0087195	0 29 48·53	1·000013	0·0000054	7·9404862	7·9404916

The first approximation may be immediately obtained by the equation

$$\log(\cot F - \cot E_0) = \log\left(\frac{e}{\theta_0}\right) + \log \operatorname{cosec} F = \log\left(\frac{e}{\theta_0}\right) - \log \sin F.$$

Let the number to the 6th decimal be N, and we get

$$\cot E_0 = -N + \cot F.$$

Computing now with the value E_0, that of F_0, $\delta F = F - F_0$, becomes a known value, and by means of Table II. the correction of E is directly obtained by the equation

$$\log \delta E = \log \delta F + \log (1 - e \cos E)^{-1} = \log \delta F + \log (1 + e \cos E).$$

II. log $(1 - e \cos E)^{-1}$.

E=	10°	20°	30°	40°	50°	60°	70°	80°	90°
☿	0·09826	0·09323	0·08197	0·07443	0·06157	0·04711	0·03166	0·01579	0·00000
♀	0·00293	0·00280	0·00257	0·00228	0·00192	0·00148	0·00101	0·00052	0·00000
♁	0·00723	0·00690	0·00635	0·00562	0·00471	0·00366	0·00250	0·00126	0·00000
♂	0·05317	0·05659	0·04639	0·04079	0·03396	0·02619	0·01816	0·00891	0·00000
♃	0·02112	0·02014	0·01852	0·01634	0·01368	0·01060	0·00723	0·00365	0·00000
♄	0·02463	0·02347	0·02158	0·01904	0·01589	0·01232	0·00840	0·00424	0·00000
♅	0·02040	0·01944	0·01787	0·01628	0·01320	0·01023	0·00748	0·00353	0·00000
♆	0·00375	0·00371	0·00329	0·00292	0·00362	0·00190	0·00129	0·00066	0·00000